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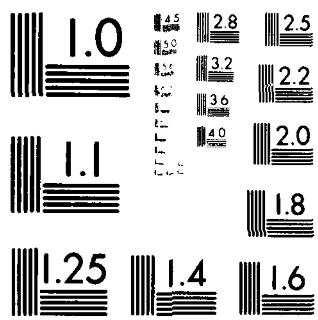


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MEMORANDUM REPORT ARBRL-MR-03006

(Supersedes IMR No. 653)

# RESOURCE EXPENDITURES AND EXPECTED TIME TO OBTAIN BINARY OBJECTIVES


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March 1980




US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>Two equations are derived which predict the expected number of attempts and time to achieve objectives which have only two states: success and failure.<br>These equations can be used to evaluate the relative merits of strategies associated with munition mixes and/or methods of sequentially delivering them, e.g., adjusted fire, change of warhead type, etc.<br>Several examples are discussed to familiarize the reader with potential applications. | | |

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# I. INTRODUCTION

This report describes the derivation and utilization of two equations which predict the number of attempts and the time which can be expected to achieve objectives which have but two states, success and failure. This report refers to these types of objectives as binary.

The equations are merely generalizations of the solution to the simplist of situations where the expected number of attempts, $E_N$, is equal to the inverse of the probability of success of each attempt, P; i.e., $E_N = 1/P$. As will be seen, this degenerate case is premised on total independence of attempts and a highly restrictive condition that the probability of success of each attempt is constant and equal to all others.

The equations derived herein allow for the analysis of a much broader class of problems, including interdependence and varying probabilities of success. However, the condition that the objective of an attempt be binary is still required. Since binary objectives are common to systems analyses, this requirement is not overly restrictive. For example the objectives assumed are nearly all duel and battlefield simulations of tank encounters are binary since the tanks are either assumed killed* or unaffected by an individual threat attempt.

Section II describes the actual derivation of the equations whereas Section III describes, by example, potential applications of the equations.

Section IV is a short summary intended to alert the reader to the advantages and restrictions of the equations discussed in the previous sections.

# II. DERIVATIONS

This section describes the derivation of two equations predicting the expected number of attempts and the expected time to achieve binary objectives.

The first derivation described is that for predicting the expected number of attempts to success. This will be in considerably more detail than the time to success derivation, which can be viewed as a simple excursion from the first. As an aside, an algorithm is provided which results in the median number of attempts required and median time to success. This is useful information in that it allows the analyst to determine the 50% point of a given strategy.

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*Mobility, firepower, and/or catastrophic kill.*

## A. Expected Number of Attempts to Achieve Binary Objectives

This is handled as a standard expected value problem which, by definition, merely sums all the statistically possible values weighted by the probability that the values will occur, i.e.:

$$(1) \quad E_N = \sum_{\forall i} i\tilde{P}(i)$$

where $E_N \equiv$ expected value of attempts

$i \equiv$ the number of attempts to success

$\tilde{P}(i) \equiv$ the probability that the "ith" attempt will be the first to succeed.

Note the $\tilde{P}(i)$ is just the probability that the ith attempt succeeds given that all (i - 1) attempts failed multiplied by the probability that all (i - 1) attempts would fail. If we define

$P(i) \equiv$ the probability that at least one success would result in "i" attempts,

then [1 - P(i)] is the probability that all i attempts failed. Since

$$P(i) = P(i - 1) + \tilde{P}(i) ,$$

it follows that

$$(2) \quad \tilde{P}(i) = P(i) - P(i - 1) .$$

This will prove to be a very important relation since it uses the simplest of probability calculations and its form will allow the contraction of several series later in the derivation.

Associated with each attempt is a probability that the attempt will succeed given that all previous attempts failed. Often these probabilities are related to immediately preceding or subsequent attempts. For example, if an artillery piece had a firing strategy consisting of three rounds to register and the rest for effect, the first three attempts would be related in a different manner than the remainder.

To assure that these similarities can be used, the series of Equation (1) are rewritten as a series of series, i.e.,

$$(3) \quad E_N = \sum_{i=1}^{N_1} i\tilde{P}(i) + \sum_{i=N_1+1}^{N_1+N_2} i\tilde{P}(i) + \ldots .$$

where $N_i$ is the number of attempts which have been identified as having some similarities in their probabilities of success with other attempts in the "ith" subsequence. Since one can always set $N_i = 1$, no loss of generality occurs.

It will prove convenient to further focus on the individual subsequences. Therefore, define

$P_K(i) \equiv$ the probability that at least one success would result if the first "i" attempts were made in the "Kth" subsequence given that all attempts prior to the "K" subsequence failed.

To provide a glimpse of where this is leading, consider two subsequences consisting of three and five attempts, respectively; i.e., the first three attempts are potentially related as are the last five attempts. Then

$$\tilde{P}(6) = [(P_2(3) - P_2(2)]*[1 - P_1(3)] \ .$$

Note that the first bracketed term uses the relationship of Equation (2) whereas the second bracketed term is merely the probability that all attempts in the first subsequence failed to succeed.

Returning to Equation (3) with our newly defined term and a slight rewrite to clarify the range of indices

$$(4) \quad E_N = \sum_{i=1}^{N_1} iP_1(i) + \sum_{i=1}^{N_2} [i + N_1]*[P_2(i) - P_2(i - 1)]*[1 - P_1(N_1)]$$

$$+ \sum_{i=1}^{N_3} [i + N_1 + N_2]*[P_3(i) - P_3(i - 1)]*[1 - P_1(N_1)]*[1 - P_1(N_2)]$$

$$+ \ . \ . \ . \ .$$

We define

$S_K \equiv \sum_{i=1}^{K} N_i$ which is the total number of attempts prior to the "j+1" subsequence

$\Pi_K \equiv [1 - P_1(N_1)] * \ldots * [1 - P_K(N_K)]$ which is the probability of failing through $S_K$ attempts. Note: $\Pi_o = 1$.

Then Equation (4) can be written as

$$(5) \qquad E_N = \sum_{\forall K} \sum_{i=1}^{N_K} [i + S_{K-1}]*[P_K(i) - P_K(i - 1)]*\Pi_{K-1} \; .$$

Equations (1) through (5) require that all statistically possible values of i be included. However, often physical constraints/inclinations limit the number of attempts possible. For example, a tank containing 40 rounds cannot make more than 40 attempts even though there is a finite probability associated with success after 40 attempts.

One method of circumventing this problem is to assume that whatever strategy was used during the admissible (physically possible) number of attempts would be used again and again, i.e., is cyclic. In the tank example, this is akin to assuming it will reload and engage in the same manner.

Employing this method, Equation (5) becomes

$$(6) \qquad E_N = \sum_{j=0}^{\infty} \sum_{K=1}^{t} \sum_{i=1}^{N_K} [i + S_{K-1} + j*S_t]*[P_K(i) - P_K(i - 1)]*\Pi_{K-1}*(\Pi_t)^j$$

"t" $\equiv$ the total number of subsequences considered in the admissible sequence.

Note that $S_t$ is the total number of admissible attempts.

We are now ready to simplify the equation into a form that justifies the expenditure of time to this point in the derivation. To accomplish this, the following relationships are used:

(7) $$\sum_{i=0}^{\infty} (\Pi_t)^i = (1 - \Pi_t)^{-1}$$

(8) $$\sum_{j=0}^{\infty} j(\Pi_t)^j = \Pi_t*(1 - \Pi_t)^{-2}$$

(9) $$\sum_{i=1}^{N_K} \left[P_K(i) - P_K(i - 1)\right] = P_K(N_K)$$

(10) $$\sum_{i=1}^{N_K} i*\left[P_K(i) - P_K(i - 1)\right] = N_K P_K(N_K) - \psi_K$$

where $\psi_K \equiv \sum_{i=1}^{N_{K-1}} P_K(i)$ ,

Inserting these relationships into Equation (6) results in

(11) $$E_N = \sum_{K=1}^{t} \left\{\Pi_{K-1}\left[\frac{S_K*P_K(N_K) - \psi_K}{1 - \Pi_t} + \frac{S_t*P_K(N_K)\Pi_t}{(1 - \Pi_t)^2}\right]\right\}.$$

A final simplification is made by including the following observations:

(12) $$P_K(N_K)\Pi_{K-1} = \Pi_{K-1} - \Pi_K$$

(13) $$\sum_{K=1}^{t} (\Pi_{K-1} - \Pi_K) = 1 - \Pi_t$$

(14) $$\sum_{K=1}^{t} S_K(\Pi_{K-1} - \Pi_K) = -S_t\Pi_t + \sum_{K=1}^{t} N_K\Pi_{K-1}$$

These, when inserted into Equation (1), precipitate the desired equation.

(15) $$E_N = \sum_{K=1}^{t} \frac{\Pi_{K-1}}{(1 - \Pi_t)} (N_K - \psi_K) .$$

This equation contains a single form of hit probability, $P_K(i)$, which is usually the simplest to construct, is finite and if the subsequences are chosen with discretion, allows for maximum utilization of intra-sequence simulation.

B. Expected Time to Achieve Binary Objectives

This problem is handled in the same way as the preceding derivation. Therefore, many of the intervening steps will be skipped. Using the same nomenclature, we begin by constructing the basic function to be analyzed, i.e.,

$$(16) \qquad E_T = \sum_{V_K} \sum_{i=1}^{N_K} \left[(i - 1)\,\Delta_K + \alpha_K + T_{K-1}\right]\left[P_K(i) - P_K(i - 1)\right] * \Pi_{K-1}$$

where $\Delta_K \equiv$ the time between attempts in the "K" subsequence

$\alpha_K \equiv$ the time between the $S_{K-1}$ attempt and the first attempt of subsequence "K"

$T_K \equiv$ the time to complete $S_K$ attempts

It should be noted that there is an implicit assumption made when defining $\Delta_K$, i.e., that the intra-subsequence time intervals are constant. This could greatly influence how the sequence is sectioned, since this condition must hold if the following derivation is to be valid. Again, since $N_i$ can be set equal to unity, generality is not lost, but caution is advised.

Circumventing the finite attempt constraint in a similar manner as previously discussed, Equation (16) becomes

$$(17) \qquad E_T = \sum_{j=0}^{\infty} \sum_{K=1}^{t} \sum_{i=1}^{N_K} \left[(i - 1)\,\Delta_K + \alpha_K + T_{K-1} + (j * T_t)\right] \cdot$$

$$\left[P_K(i) - P_K(i - 1)\right] * \Pi_{K-1} * (\Pi_t)^j \ .$$

Employing the relationships depicted by Equations (7), (8), (9), and (10), this can be reduced to

$$(18) \qquad E_T = \sum_{K=1}^{t} \Pi_{K-1} \left\{ \left[\frac{T_K P_K(N_K) - \Delta_K \psi_K}{(1 - \Pi_t)}\right] + \left[\frac{T_t \Pi_t P_K(N_K)}{(1 - \Pi_t)^2}\right] \right\}$$

with the aid of Equation (12) and the observation that

$$\sum_{K=1}^{t} T_K(\Pi_{K-1} - \Pi_K) = -\Pi_t T_t + \sum_{K=1}^{t} \Pi_{K-1}\tau_K$$

where $\tau_K \equiv T_K - T_{K-1}$ .

Equation (18) takes the form

$$(19) \qquad E_T = \sum_{K=1}^{t} \frac{\Pi_{K-1}}{1 - \Pi_t} (\tau_K - \Delta_K\psi_K)$$

which has the same basic benefits as does Equation (15), i.e., simple and finite.

Equations (15) and (19) represent the average number of attempts required and the average time to successfully achieve binary objectives. This should not be confused with the median number of attempts and associated time to achieve the same.

C. Median Values

Median values are those for which 50% of the time, success would have been achieved in attempts/time less than or equal to the value. Although median values are of interest in systems analyses, they contain less information than expected values and therefore are less useful. However, since the mathematical framework is already at hand, the following algorithm can be used to determine the median values at the same time one is solving Equations (15) and (19).

- While calculating $\Pi_K$ required in Equations (15) and (19) test for

$$(\Pi_K - 0.5)*(\Pi_{K-1} - 0.5) \overset{?}{\leq} 0$$

- Having found K such that the preceding inequality is satisfied, monitor the calculations used in $\psi_K$ to find i such that

$$\left\{P_K(i) - \left[1 - \frac{0.5}{\Pi_{K-1}}\right]\right\} * \left\{P_K(i - 1) - \left[1 - \frac{0.5}{\Pi_{K-1}}\right]\right\} \leq 0 \ .$$

- The value of i satisfying this inequality when added to $S_K$ represents the median number of attempts to success, and the time associated with making $i + S_K$ attempts represents the median time to success.

## III. EXAMPLES

This section discusses several example applications of Equations (15) and (19). As will become evident the examples become progressively more complex.

E1. Constant Probabilities. Every so often, the sun shines, flowers grow, and the problem at hand is one where each attempt has a constant independent equal probability of success. A tank with no fixed or variable bias attempting to hit another tank, and a man trying to "flip heads" on a coin represent two such situations.

In any case, let "P" represent the probability of success associated with each attempt, then it can easily be shown that

$$P_K(N_K) = 1 - (1 - P)^{N_K}$$

$$\psi_K = N_K - \frac{1 - (1 - P)^{N_K}}{P}$$

$$\tilde{\Pi}_K = (1 - P)^{S_K} .$$

Substitution into Equation (15) results in

$$N = \sum_{K=1}^{t} \left[ \frac{\Pi_{K-1}}{(1 - \Pi_t)} \left\{ N_K - N_K - \frac{[1 - (1 - P)^{N_K}]}{P} \right\} \right]$$

$$= \sum_{K=1}^{t} \frac{\Pi_{K-1}}{(1 - \Pi_t)} \frac{P_K(N_K)}{P}$$

which by Equations (12) (13) can be reduced to

$$(20) \qquad E_n = \frac{1}{P}$$

which is not the most surprising result ever derived. Now assume that the time between events is constant, i.e., $\Delta_i = \Delta$, $\alpha_i = \Delta$. Then,

$$\tau_K = (N_{K-1})\Delta + \Delta$$

and Equation (19) becomes

$$(21)\qquad E_T = \sum_{K=1}^{t} \frac{\Pi_{K-1}}{1 - \Pi_t}\left\{(N_K - 1)\Delta + \Delta - \Delta\left[N_K - \frac{1 - (1 - P)^{N_K}}{P}\right]\right\}$$

$$= \frac{\Delta}{P}\ .$$

Implicit in this example is the assumption that the time between start and the first attempt is $\Delta$. One can eliminate that by assuming $\alpha_1 = 0$, $\alpha_{i\neq 1} = \Delta$ where upon, for large $S_K$

$$(22)\qquad E_T = (\frac{1}{P} - 1)\Delta\ .$$

For a specific example where the probability of success associated with each attempt is 0.2 and the time between attempts is 10 s

$$E_N = \frac{1}{0.2} = 5 \text{ attempts}$$

$$E_T = (\frac{1}{0.2} - 1)*10 = 40 \text{ s.}$$

Using the algorithm of Section II, it is easily shown that for this example, the *median* number of attempts is 3 and *median* time is 20 s.

E2. Constant Probabilities within Subsequence. Sometimes the probability of success is constant for each attempt in a subsequence, but assumes a different value for each subsequence. A firepower system with no bias delivering two distinct types of rounds to destroy a target is such an animal.

For this example, assume $P_K$ is the probability of success for each attempt in subsequence "K." Then

$$P_K(N_K) = 1 - (1 - P_K)^{N_K}$$

$$\psi_K = N_K - \left[\frac{1 - (1 - P_K)^{N_K}}{P_K}\right]$$

$$\Pi_K = \prod_{i=1}^{K} (1 - P_i)^{N_i}\ .$$

Substitution with Equation (15) leads to

$$(23) \qquad E_N = \sum_{K=1}^{T} \frac{\Pi_{K-1}}{(1 - \Pi_T)} \left[ \frac{1 - (1 - P_K)^{N_K}}{P_K} \right]$$

As a specific example assume that a firepower system has two rounds of type A and two rounds of type B ammunition. If A has a probability of success of 0.4 and B, 0.3, which sequence of delivery is best--ABAB, BABA, AABB, or BBAA? Equation (23) provides the following results by setting t = 4 and $N_i = 1$:

$E_N$: ABAB = 2.8

$E_N$: BABA = 2.9

$E_N$: AABB = 2.7

$E_N$: BBAA = 3.0

Thus sequence AABB is a marginally better strategy if delivery of resource depletion per success is the criteria. Not surprising!

On the other hand, if time to success were important and if there were an additional time burden associated with type A rounds, the result is not so obvious. Assume that type A, being bulky, requires 7 s to load and fire whereas type B requires only 5 s. Then,

$E_T$: ABAB = 17.0 s

$E_T$: BABA = 16.8 s

$E_T$: AABB = 17.1 s

$E_T$: BBAA = 16.8 s

which leads to the conclusion that both BABA and BBAA are the better strategies.

However, if one assumes that the first round is already chambered (i.e., $\alpha_1 = 0$) one could conclude that ABAB is the best strategy since

$E_T$: ABAB = 10 s

$E_T$: BABA = 11.8 s

$E_T$: AABB = 10.1 s

$E_T$: BBAA = 11.8 s

As this example illustrates, it is rather important to use the equation specifically applicable to the measure of interest.

E3. Intradependent Attempts. Unfortunately, it is often the case that the probability of success of each attempt in a particular sequence is implicitly dependent on the success or failure of previous attempts in the subsequence. This is the case with most firepower systems which are plagued with both round-to-round and occasion-to-occasion errors. It is the latter which causes intradependence whereas changing munitions and/or aimpoints causes the interdependence of attempts.

Assume one is analyzing a firepower system which has an error budget such that random, variable bias, and fixed bias error distributions are known for the range in question. Also, assume that the conditional kill probability can be reasonably estimated for the targets of interest. Using the following definitions, we set up the solution. Let:

$x,y \equiv$ spacial variables indicating the impact point of the projectile in the plane of the target. The plane may be defined as vertical or horizontal depending on weapon type, e.g., tank, artillery.

$\eta_x, \eta_y \equiv$ variable describing the system bias consisting of both a fixed and variable bias

$\sigma_{R_x}^2, \sigma_{R_y}^2 \equiv$ the variance of the random errors, in the x and y direction

$\sigma_{\eta_x}^2, \sigma_{\eta_y}^2 \equiv$ the variance of the variable biases

$\mu_x, \mu_y \equiv$ the fixed bias in the x and y direction

$K_i(x,y) \equiv$ the conditional probability that the target will be defeated if the projectile impacts at point x,y in the "ith" sequence.

Assume that all error sources have normal distributions. Further define

$$\rho(x,y:\eta_x,\eta_y) = \frac{\exp -\left[\frac{1}{2}\frac{(x-\eta_x)^2}{\sigma_{R_x}^2} + \frac{(y-\eta_y)^2}{\sigma_{R_y}^2}\right]}{\Pi\sigma_{R_x}\sigma_{R_y}}$$

$$\bar{\rho}_i(\eta_x,\eta_y) = \iint_{-\infty}^{\infty} K(x,y)\rho(x,y:\eta_x,\eta_y)\, dx\, dy$$

which is the probability of defeating a target with an attempt given specific values for $\eta_x, \eta_y$. Note that for many munitions against hard targets, such as tanks, the bounds of this integration are limited to the projection of the target on the target plane. This occurs because $K_i(x,y) = 0$ when the projectile misses these target types.

$$(24) \quad P_K(N_K) = 1 - \iint_{-\infty}^{\infty} \left[1 - \bar{\rho}_K(\eta_x,\eta_y)\right]^{N_K} \rho(\eta_x,\eta_y:\mu_x,\mu_y)\, d\eta_x\, d\eta_y$$

$$\psi_K(N_K) = N_K - \iint_{-\infty}^{\infty} \left\{ \frac{1 - \left[1 - \bar{\rho}_K(\eta_x,\eta_y)\right]^{N_K}}{\bar{\rho}_K(\eta_x,\eta_y)} \right\} \rho(\eta_x,\eta_y:\mu_x,\mu_y)\, d\eta_x\, d\eta_y \; .$$

When these are substituted into Equation (13), the expected number of rounds used and time to defeat the target are easily calculated.

As a specific example, assume we are interested in whether adjusting fire is advantageous in a system whose error budget is

$$\sigma_{R_x} = \sigma_{R_y} = 0.6 \text{ mrad}$$
$$\sigma_{\eta_x} = \sigma_{\eta_y} = 0.4 \text{ mrad}$$
$$\mu_x = \mu_y = 0.0 \text{ mrad} \; .$$

As a baseline case, we assume no adjustment and decree that if the target is not hit in 10 rounds, firing ceases, i.e., $S_T = 10$. The range is arbitrarily chosen to be 1500 m and 3000 m, the target is a 23x23 panel and the objective is to hit the panel.

Under these conditions

| | 1500 m | 3000 m |
|---|---|---|
| $P_1(10)$ | = 0.99 | = 0.805 |
| $\psi_1(10)$ | = 7.92 | = 4.799 |

$$\therefore \quad E_\eta = \frac{10 - \psi_1(10)}{P_1(10)}$$

| | 1500 m | 3000 m |
|---|---|---|
| $E_\eta$ | = 2.1 | = 6.5 |

Now we will evaluate the option of adjusting fire after each round assuming that it can be done perfectly. Since the sample size is limited, much of the error observed and corrected for is random rather than the bias which we wish to eliminate. It can be shown that the standard deviation of the bias error reduces from its value to the value of the random error divided by the square root of the sample size. Since the strategy in this example is to use information only on the preceding round, the sample is 1.

Under these conditions

| 1500 m | 3000 m |
|---|---|
| $P_1(1) = 0.51$ | $= 0.164$ |
| $\psi_i(1) = 0$ | $= 0$ |
| $P_{i>1}(1) = 0.402$ | $= 0.122$ |
| $E_\eta = 2.2$ | $= 7.7$ |

Hence, in this example, closed-loop fire control actually degrades the system even under the assumption that everything is done perfectly, i.e., zero measurement errors.

## IV. SUMMARY

This final section merely reiterates the caveats annunciated previously and summarizes the results. The caveats noted are

- Binary Objectives. For Equations (15) and (19) to be valid the attempts being analyzed can have only two outcomes, total success or total failure. Although this represents a large class of problems, there are also many for which this condition does not hold. For example, consider a pugilist who is attempting to KO his opponent. Although an individual blow fails, it may sufficiently condition the opponent to enable a lesser blow to be successful. Thus his attempt, although not successful, did affect the objective, i.e., was not a total failure.
- Expected Versus Median Values. Equations (15) and (19) are not median values and therefore caution is advised in interpreting their results. Median values can be simultaneously found by use of the algorithm described in Section II.
- Expected Attempts Versus Time. As examples of Section III illustrate, depending on the measure considered important, use of Equation (15) versus (19) can lead to quite different conclusions. Thus caution is advised in inferring the expected time from expected attempts and vice versa.

This report discussed the derivation to two equations which when used with discretion may significantly reduce the complexity and time associated with determining the expected value of resources depletion and time to success associated with achieving binary objectives.

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